How to best use these slides…
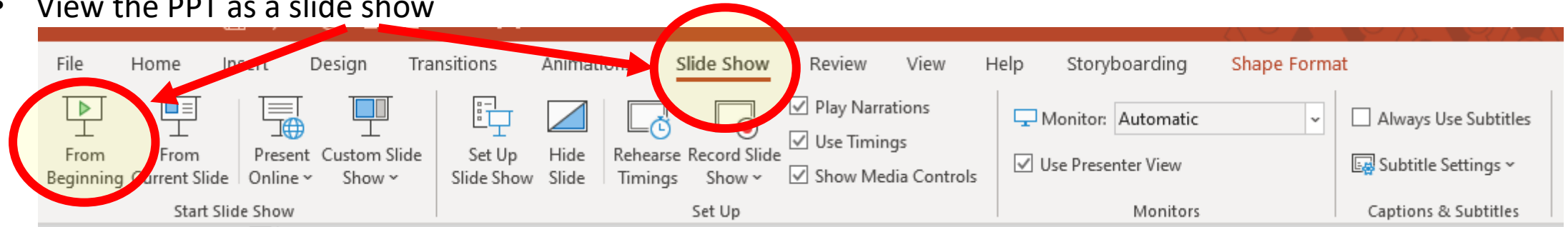
- View the PPT as a slide show

- Then click through every step

  - Mouse clicks will advance the slide show

  - Left/right arrow keys move forward/backward

  - Mouse wheel scrolling moves forward/backward

- When a question is posed, stop and think it through, try to answer it yourself before clicking

- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.3b

Dividing Rational Expressions

Today you will:

- Divide rational expressions

- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational expression, p. 376
- Simplified form of a rational expression, p. 376

Prior:
- Fractions and fraction arithmetic
- Polynomials
- Domain
- Equivalent expressions
- Reciprocal

Review/Recap from yesterday

- *Rational Expression*:

    - One polynomial divided by another

    - In other words, a fraction with a polynomial on top and another on the bottom

    - $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both non-zero polynomials

- Fraction Arithmetic - Multiplying

    1. Simplify each fraction

    2. Multiply across

    3. Simplify the result

    Alternatively you can:

    1. Multiply across first

    2. Then simply the result

    But sometimes it helps to clean up before multiplying across…

- Simplifying

    - Cancelling means "dividing out common **factors**"

    - Factor before cancelling … always

    - This mean you **CANNOT** cancel in situations like this: $\frac{x+3}{x}$ because in the numerator x is a term not a factor

Today we are going to divide *Rational Expressions*

…and we are going to use a trick to make it much easier!

The trick … How can you change a division problem into a multiplication problem?

- Multiply by the reciprocal!

- Err … what does that mean?

- What is the reciprocal of $\frac{1}{2}$? It is $\frac{2}{1}$ or 2.

- The reciprocal of a fraction is the fraction flipped on its head. ☺

- …switch the numerator and denominator

Example: Perform the following division problem: $\frac{7}{x+1} \div \frac{x+2}{2x-3}$

$$\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)} = \frac{14x-21}{x^2+3x+2}$$

→ This makes division problems with rational expressions much easier!

Find the quotient $\dfrac{7x}{2x-10} \div \dfrac{x^2-6x}{x^2-11x+30}$.

SOLUTION

$$\dfrac{7x}{2x-10} \div \dfrac{x^2-6x}{x^2-11x+30} = \dfrac{7x}{2x-10} \cdot \dfrac{x^2-11x+30}{x^2-6x}$$ Multiply by reciprocal.

$$= \dfrac{7x}{2(x-5)} \cdot \dfrac{(x-5)(x-6)}{x(x-6)}$$ Factor.

$$= \dfrac{7x(x-5)(x-6)}{2(x-5)(x)(x-6)}$$ Multiply. Divide out common factors.

$$= \dfrac{7}{2}, \quad x \neq 0, \, x \neq 5, \, x \neq 6$$ Simplified form

Find the quotient $\dfrac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$.

SOLUTION

$$\dfrac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \dfrac{6x^2 + x - 15}{4x^2} \cdot \dfrac{1}{3x^2 + 5x} \qquad \text{\textcolor{red}{Multiply by reciprocal.}}$$

$$= \dfrac{(3x + 5)(2x - 3)}{4x^2} \cdot \dfrac{1}{x(3x + 5)} \qquad \text{\textcolor{red}{Factor.}}$$

$$= \dfrac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}} \qquad \text{\textcolor{red}{Divide out common factors.}}$$

$$= \dfrac{(2x - 3)}{4x^3}, \; x \neq -\dfrac{5}{3} \qquad \text{\textcolor{red}{Simplified form}}$$

The total annual amount I (in millions of dollars) of personal income earned in Alabama and its annual population P (in millions) can be modeled by

$$I = \frac{6922t + 106{,}947}{0.0063t + 1}$$

and

$$P = 0.0343t + 4.432$$

where t represents the year, with $t = 1$ corresponding to 2001. Find a model M for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume $t > 0$.)

SOLUTION

To find a model M for the annual per capita income, divide the total amount I by the population P.

$$M = \frac{6922t + 106{,}947}{0.0063t + 1} \div (0.0343t + 4.432) \qquad \text{Divide } I \text{ by } P.$$

$$= \frac{6922t + 106{,}947}{0.0063t + 1} \cdot \frac{1}{0.0343t + 4.432} \qquad \text{Multiply by reciprocal.}$$

$$= \frac{6922t + 106{,}947}{(0.0063t + 1)(0.0343t + 4.432)} \qquad \text{Multiply.}$$

To estimate Alabama's per capita income in 2010, let $t = 10$ in the model.

$$M = \frac{6922 \cdot 10 + 106{,}947}{(0.0063 \cdot 10 + 1)(0.0343 \cdot 10 + 4.432)}$$

Substitute 10 for t.

$$\approx 34{,}707$$

Use a calculator.

▶ In 2010, the per capita income in Alabama was about $34,707.

Review/Recap

- *Dividing Rational Expression*:

  - Change to a multiplication problem

  - Multiply by the reciprocal!

Homework

Pg 381, #27-38